MECHANICAL WAVES

Mechanical Waves :-

Mechanical waves are those waves which require material medium for their propagation from one point to another.

⇒ Mechanical waves are also called elastic waves because their propagation depends upon the elastic properties of the medium.

⇒ The mechanical waves exist in all three states of matter : solid, liquid and gas.

⇒ The mechanical waves may be transverse or longitudinal in nature.

eg:- Sound waves, waves on the surface of water, seismic waves, waves in pipes, waves in strings, etc.

Velocity of sound in a medium (By the method of dimensions):-

It is observed that the velocity (v) of sound in a medium depends on the elasticity (E) of the medium and its density (ρ). Thus,

$$v \propto E^{x} \text{ ——— } (1)$$

$$\& \quad v \propto \rho^{y} \text{ ——— } (2)$$

where; x & y are indices to be determined.

Combining relations (1) & (2), we get

$$v \propto E^{x} \cdot \rho^{y}$$

$$\therefore v = k E^{x} \cdot \rho^{y} \text{ ——— } (3)$$

where; 'k' is a dimensionless proportionality constant.

Now,

Dimension of 'v' $= [M^0LT^{-1}]$

Dimension of 'E' $= [ML^{-1}T^{-2}]$

Dimension of 'ρ' $= [ML^{-3}T^0]$

Substituting the corresponding dimensions in eqⁿ (3), we get

$$[M^0LT^{-1}] = [ML^{-1}T^{-2}]^x [ML^{-3}T^0]^y$$

$$\Rightarrow [M^0LT^{-1}] = [M^xL^{-x}T^{-2x}][M^yL^{-3y}T^0]$$

$$\Rightarrow [M^0LT^{-1}] = [M^{x+y}L^{-x-3y}T^{-2x}]$$

Equating the corresponding indices, we get

$$x+y=0$$

$$\Rightarrow y=-x \quad \text{---} (4)$$

And,

$$-2x=-1$$

$$\Rightarrow x=\frac{1}{2}$$

Thus from (4),

$$y=-\frac{1}{2}$$

Substituting the values of x & y in eqⁿ (3), we get

$$v = kE^{1/2}\cdot\rho^{-1/2}$$

$$\Rightarrow v = k \cdot \frac{E^{1/2}}{\rho^{1/2}}$$

$$\Rightarrow v = k \left(\frac{E}{\rho}\right)^{1/2}$$

$$\therefore v = k\sqrt{\frac{E}{\rho}}$$

From Mathematical analysis, $k = 1$.

$$\therefore \boxed{v = \sqrt{\frac{E}{\rho}}}$$

This is the required expression for the velocity of sound in a medium.

\# In case of a liquid/gas, modulus of elasticity is the bulk modulus of elasticity, i.e. $E = B$.

$$\therefore \boxed{v = \sqrt{\frac{B}{\rho}}}$$

\# In case of a solid rod/bar, modulus of elasticity is the Young's modulus of elasticity, i.e. $E = Y$.

$$\therefore \boxed{v = \sqrt{\frac{Y}{\rho}}}$$

Newton's formula for velocity of sound in gas:-

The velocity of sound waves in a gas is given by,

$$v=\sqrt{\frac{B}{\rho}} \qquad (1)$$

where; B = Bulk modulus of elasticity of the gas
& ρ = density of the gas.

Newton assumed that when sound waves propagate through air, compressions and rarefactions are formed. He argued that the change in pressure and volume in these regions is so slow that the heat produced during compression is given to the surrounding air while the heat lost during rarefaction is gained from the surrounding gas. So, the temperature of the medium remains constant. Thus, according to Newton, the propagation of sound waves through air is an isothermal process.

For an isothermal process,

$$PV = \text{constant} \qquad (2)$$

where; P = pressure of the gas
& V = volume of the gas.

On differentiating eq^{n} (2), we get

$$PdV + VdP = 0$$

$$\Rightarrow PdV = -VdP$$

$$\Rightarrow P = -\frac{VdP}{dV}$$

$$\Rightarrow P = \frac{dP}{-\left(\frac{dV}{V}\right)}$$

$$\Rightarrow P = B \qquad (3)$$

where; $\frac{dP}{-\left(\frac{dV}{V}\right)} = B$, the bulk modulus of air.

Using (3) in (1), we get

$$\boxed{v = \sqrt{\frac{P}{\rho}}} \text{ ——— (4)}$$

This is Newton's formula for the velocity of sound in a gas.

At NTP,

$$P = 760 \text{ mm of Hg}$$
$$= 1.01 \times 10^5 \text{ N/m}^2$$

$$\& \ \rho = 1.293 \text{ kg/m}^3$$

Thus, from eqn (4),

$$v = \sqrt{\frac{1.01 \times 10^5}{1.293}}$$

$$\Rightarrow v \simeq 280 \text{ m/s}$$

According to Newton's formula, the velocity of sound in air at NTP is 280 m/s. However, the velocity of sound in air at NTP is found to be 332 m/s, which is quite greater than the value calculated by Newton's formula. This is due to some discrepancies in Newton's assumption and it needs to be corrected.

Discrepancy of Newton's Formula – Laplace Correction :-

More than one century later the Newton's derivation, Laplace in 1816, corrected this formula. He pointed out that Newton's assumption of propagation of sound in gas as a isothermal process was a mistake!

According to Laplace, the process of compression & rarefaction occur so rapidly that neither heat is transferred to the surroundings during compression nor heat is taken from the surrounding during rarefaction. So, the temperature in different regions does not remain constant. Thus, the propagation of sound waves through air is an adiabatic process.

For an adiabatic process,

$$PV^{\gamma} = \text{constant} \quad \text{——— (5)}$$

where; $\gamma = C_p/C_v$; the ratio of molar heat capacities of air.

On differentiating eq^n (5), we get

$$P(\gamma V^{\gamma-1} dV) + V^{\gamma} dP = 0$$

$$\Rightarrow \gamma P V^{\gamma-1} dV + V^{\gamma} dP = 0$$

Dividing both sides by $V^{\gamma-1}$, we get

$$\frac{\gamma P V^{\gamma-1} dV}{V^{\gamma-1}} + \frac{V^{\gamma} dP}{V^{\gamma-1}} = 0$$

$$\Rightarrow \gamma P dV + V dP = 0$$

$$\Rightarrow \gamma P\,dV = -V\,dP$$

$$\Rightarrow \gamma P = -\frac{V\,dP}{dV}$$

$$\Rightarrow \gamma P = -\frac{dP}{\left(\frac{dV}{V}\right)}$$

$$\Rightarrow \gamma P = B \quad \text{——— (6)}$$

Using (6) in (1), we get

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}} \longleftarrow (7)$$

This is corrected Newton's formula the velocity of sound in gas.

At NTP,

$$P = 760\,\text{mm of Hg}$$
$$= 1.01 \times 10^5\ N/m^2$$

$$\rho = 1.293\ kg/m^3$$

$$\& \ \gamma = 1.4$$

Thus, from eqn (7)

$$v = \sqrt{\frac{1.4 \times (1.01 \times 10^5)}{1.293}}$$

$$\therefore v = 331.2\ m/s.$$

This result is in close agreement with the experimental value. Thus, Laplace's formula gives the correct value of velocity of sound in air.

Factors Affecting the velocity of sound in Gas :-

The velocity of sound in a gas is given by,

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}} \quad \text{---- (1)}$$

where; P = pressure of the gas
ρ = density of the gas
$\gamma = \frac{C_P}{C_V}$, the ratio of molar heat capacities of the gas.

- For 'n' moles of gas, the equation of state is

$$PV = nRT \quad \text{---- (2)}$$

But,

$$n = \frac{m}{M}$$

where; m = given mass of the gas
M = molar mass of the gas.

Thus, eqn. (2) becomes

$$PV = \frac{m}{M} RT$$

$$\Rightarrow \frac{PV}{m} = \frac{RT}{M}$$

$$\Rightarrow \frac{P}{\left(\frac{m}{V}\right)} = \frac{RT}{M}$$

$$\Rightarrow \frac{P}{\rho} = \frac{RT}{M} \quad \text{---} (3) \qquad \left[\because \rho = \frac{m}{V}\right]$$

Using (3) in (1), we get

$$\boxed{v = \sqrt{\frac{\gamma RT}{M}}} \quad \text{---} (4)$$

Now, let's discuss the factors affecting the velocity of sound in gas.

(01) Effect of Pressure :-

The increase in pressure of the gas causes an equivalent increase in the density of the gas so that the ratio of P/ρ is always constant. Hence from (1), the velocity of sound in gas is independent of the pressure of the gas at constant temperature.

(02) Effect of density :-

From eqⁿ (1), we have

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

For any two gases (say Gas 1 and Gas 2) having same atomicity (γ = constant) and same pressure P,

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}}$$

$$\& \quad v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

Thus,

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{\gamma P}{\rho_2}}}{\sqrt{\frac{\gamma P}{\rho_1}}}$$

$$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{\gamma P}{\rho_2}} \times \sqrt{\frac{\rho_1}{\gamma P}}$$

$$\Rightarrow \boxed{\frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}}}$$

$$\Rightarrow \boxed{v \propto \frac{1}{\sqrt{\rho}}}$$

Thus, the velocity of sound in a gas is inversely proportional to the square root of the density of the gas, at constant pressure.

(03) Effect of Humidity :-

The density of water vapour is smaller than that of dry air. Thus, the presence of moisture reduces the density of air. So, $\rho_{moist\ air} < \rho_{dry\ air}$. Since $v \propto \frac{1}{\sqrt{\rho}}$, hence the velocity of sound is greater in moist air than in dry air.

Greater the humidity of air, higher is the velocity of source.

(04) Effect of Temperature :-

From eq^n (4), it is clear that the velocity of of sound in gas is directly proportional to the square root of the absolute temperature of the gas.

i.e. $\boxed{v \propto \sqrt{T}}$

If v_1 and v_2 be the velocities of a gas at temperatures T_1 & T_2 respectively, then

$$\boxed{\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}}$$

(05) Effect of Wind:-

The velocity of sound is greater in the direction of the wind and less in the opposite direction.

The net velocity of sound is given by,

$$\boxed{V_{net} = V \pm V_w}$$

where; V = velocity of sound.
V = velocity of wind.

(06) Effect of frequency, wavelength and amplitude:-

- The velocity of sound in air is independent of both frequency and wavelength. The velocity of sound is independent of amplitude as well.

(07) Effect of nature of gas :-

From eqⁿ (4), we have

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Thus at constant temperature T,

$$\boxed{v \propto \sqrt{\gamma}}$$

$$\& \boxed{v \propto \frac{1}{\sqrt{M}}}$$

The velocity of sound in a gas depends upon the atomicity of the gas ($v \propto \sqrt{\gamma}$) and the molecular mass of the gas ($v \propto \frac{1}{\sqrt{M}}$).